

Algebra 2 Semester 2 Exam Review

Evaluate:

1. $81^{3/4} = (\sqrt[4]{81})^3 = 3^3 = 27$

2. $16^{-5/4} = \frac{1}{16^{5/4}} = \frac{1}{(\sqrt[4]{16})^5} = \frac{1}{2^5} = \frac{1}{32}$

3. Let $r(x) = x^2 - 3$ and $s(x) = x - 6$.

Find $r(s(x))$.
 $= (x-6)^2 - 3$
 $= x^2 - 12x + 36 - 3$
 $= x^2 - 12x + 33$

Find $r(s(-1))$.
 $= (-1)^2 - 12(-1) + 33$
 $= 1 + 12 + 33$
 $= 46$

4. Let $f(x) = 16x^2$ and $g(x) = 2x^3$. Find the following:

a. $f(x) \cdot g(x) = (16x^2)(2x^3) = 32x^5$

b. $\frac{f(x)}{g(x)} = \frac{16x^2}{2x^3} = \frac{8}{x}$

c. the domains of $f \cdot g$ and $\frac{f}{g}$

↓

all Real #'s

↓

all real,
 $x \neq 0$

5. Which is an equation for the inverse of the relation $y = 4x + 2$?

$x = 4y + 2$
 $x - 2 = 4y$
 $\frac{x-2}{4} = y$

- a. $y = 2x + 4$
- b. $y = \frac{4x-2}{4}$
- c. $y = \frac{x+2}{4}$
- d. $y = \frac{x-2}{4}$

Solve the equation. Check for extraneous solutions.

6. $\frac{3x^{3/4}}{3} = \frac{192}{3}$

$x^{3/4} = 64$

$(x^{3/4})^{4/3} = (64)^{4/3}$

$x = (\sqrt[3]{64})^4$

$x = 4^4 = 256$

7. $[(3x-8)^{1/2}]^2 = 5^2$

$3x-8 = 25$

$3x = 33$

$x = 11$

8. $\sqrt{7x+15} = x+1$

$(\sqrt{7x+15})^2 = (x+1)^2$

$7x+15 = x^2 + 2x + 1$

$-7x \quad -15 \qquad \qquad -7x \quad -15$

$0 = x^2 - 5x - 14$

$0 = (x-7)(x+2)$

$x = 7, -2$

Check:

7: $\sqrt{49+15} = 7+1 \quad -2: \sqrt{49+15} = -2+1$

$\sqrt{64} = 8$

$1 \neq -1$

-2 is extraneous

9. The projected worth (in millions of dollars) of a large company is modeled by the equation $y = 241(1.04)^x$. The variable x represents the number of years since 1997. What is the projected annual percent of growth, and what should the company be worth in 2001? (4th year, so $x=4$)

1.04 growth factor = 4% growth

- a. 14%; \$293.21 million
 b. 4%; \$271.09 million
 c. 14%; \$250.64 million
 d. 4%; \$281.94 million

Year 4: $y = 241(1.04)^4 = 281.94$ million

10. A piece of equipment costs \$85,000 new but depreciates 15% per year in each succeeding year. Find its value after 10 years.

decay factor = $1 - 0.15 = 0.85$

$85,000(0.85)^{10} = \$16,734.32$

11. Evaluate without using a calculator. $\log_2 16$

$2^4 = 16$, so $\log_2 16 = 4$

12. Evaluate without using a calculator. $\log_7 \frac{1}{49}$

$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$ so $\log_7 \left(\frac{1}{49}\right) = -2$

13. Solve. $10^{3x} + 4 = 9$

$10^{3x} = 5$
 $\log_{10} 10^{3x} = \log_{10} 5$
 $3x = \frac{\log 5}{\log 10}$
 $3x = 0.699$
 $x = 0.233$

Solve the equation. Check for extraneous solutions.

14. $5 \log_3 (x) - 2 = 18$

$5 \log_3 x = 20$
 $\log_3 x = 4$
 $3^{\log_3 x} = 3^4$
 $x = 81$

check: $5 \log_3 81 - 2 = 18$
 $5 \cdot 4 - 2 = 18$
 $20 - 2 = 18$
 $18 = 18$ ✓

15. The price per person of renting a bus varies inversely with the number of people renting the bus. It costs \$12 per person if 66 people rent the bus. About how much will it cost per person if 77 people rent the bus?

$y = \frac{k}{x}$ $7k = 792$
 $12 = \frac{k}{66}$ $y = \frac{792}{x}$

$y = \frac{792}{77} = \$10.29$
 Simplify the rational expression, if possible.

16. $\frac{n^2 + 2n - 24}{n^2 - 11n + 28} = \frac{(n+6)(n-4)}{(n-7)(n-4)} = \frac{n+6}{n-7}$

Multiply the expressions. Simplify the result.

17. $\frac{(x+2)^2}{x-5} \cdot \frac{x^2-2x}{x^2-4} = \frac{(x+2)(x+2)}{(x-5)} \cdot \frac{x(x-2)}{(x-2)(x+2)}$
 $= \frac{x(x+2)}{x-5}$

Divide the expressions. Simplify the result.

18. $\frac{x^2+9x+18}{x^2-9} \div \frac{x+6}{x-6} = \frac{(x+6)(x+3)}{(x+3)(x-3)} \cdot \frac{(x-6)}{(x+6)}$
 $= \frac{(x-6)}{(x-3)}$

Perform the indicated operation(s) and simplify.

19. $\frac{4}{x+8} + \frac{1}{x-8}$ Common denominator $(x+8)(x-8)$
 $\frac{4}{x+8} \cdot \frac{(x-8)}{(x-8)} = \frac{4x-32}{(x+8)(x-8)}$
 $\frac{1}{(x-8)} \cdot \frac{(x+8)}{(x+8)} = \frac{x+8}{(x+8)(x-8)}$

$\frac{4x-32+x+8}{(x+8)(x-8)} = \frac{5x-24}{(x+8)(x-8)}$

Solve the equation. Check for extraneous solutions.

$$20. \left[\frac{x+2}{4x} - \frac{3}{2x} = \frac{1}{8} \right]$$

$$2(x+2) - 4(3) = x$$

$$2x + 4 - 12 = x$$

$$x = 8$$

check:

$$\frac{10}{32} - \frac{3}{16} = \frac{1}{8}$$

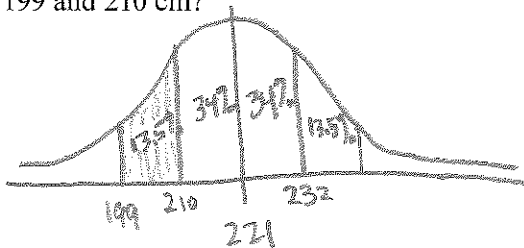
$$\frac{5}{16} - \frac{3}{16} = \frac{2}{16} = \frac{1}{8} \quad \checkmark$$

23. Open-ended Problem: Compare the quiz grades of the two algebra classes shown in the table by comparing the measures of central tendency and variation of the two data sets.

First Period	10	5	6	5	6	7	8	5	6	2	\bar{x}	med	range	σ
Second Period	2	10	10	4	2	5	1	10	9	7	6	6	9	3.46

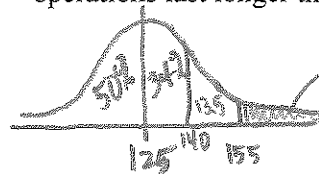
The measures of central tendency are the same but the data is more spread out for 2nd period than 1st.

24. Last year, the personal best high jumps of track athletes in a nearby state were normally distributed with a mean of 221 cm and a standard deviation of 11 cm. What is the probability that a randomly selected high jumper has a personal best between 199 and 210 cm?



13.5% will be between 199 and 210.

25. The duration of routine operations in a certain hospital has approximately a normal distribution with an average of 125 minutes and a standard deviation of 15 minutes. What percentage of operations last longer than 155 minutes?



$$100 - (50 + 34 + 13.5)$$

$$100 - 97.5$$

$$= 2.5\%$$

26. Convert 48° to radian measure.

$$48 \cdot \frac{\pi}{180} = \frac{4\pi}{15}$$

$$(k+1)(k-1)$$

$$21. \left[\frac{k}{k+1} + \frac{1}{k-1} = \frac{4k-3}{(k+1)(k-1)} \right]$$

$$k(k-1) + (k+1) = 4k-3$$

$$k^2 - k + k + 1 = 4k - 3$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)(k-2) = 0 \quad k=2$$

22. Find the distance between point $C(-8, -1)$ and point $G(6, 2)$, then find the midpoint of \overline{CG} .

$$d = \sqrt{(2 - (-1))^2 + (6 - (-8))^2} = \sqrt{3^2 + 14^2} = 14.3$$

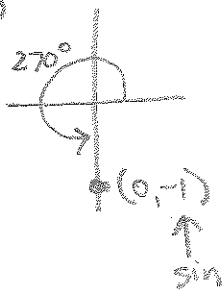
$$\text{midpoint} \left(\frac{-8+6}{2}, \frac{-1+2}{2} \right) = \left(\frac{-2}{2}, \frac{1}{2} \right) = \left(-1, \frac{1}{2} \right)$$

check: $\frac{5}{16} + \frac{1}{16} = \frac{6}{16} = \frac{3}{8}$
 $\frac{4(2)-3}{(2+1)(2-1)} = \frac{5}{1} = 5$
 $\frac{3}{8} \neq 5$

Evaluate the function without using a calculator.

27. $\sin \frac{3\pi}{2} = \sin 270^\circ$

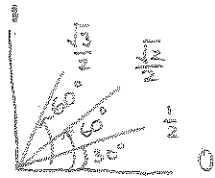
- a. $\frac{\sqrt{2}}{2}$
- b. 0
- c. -1
- d. $\frac{\sqrt{2}}{2}$



28. Without using a calculator, find the exact value of

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$$

- a. $\frac{1}{3}\pi$
- b. $\frac{1}{2}\pi$
- c. $-\frac{3}{4}\pi$
- d. $-\frac{1}{4}\pi$



$45^\circ = \frac{\pi}{4}$
 sin is negative in
 Quadrants 3 & 4.
 Ref $\angle = 45^\circ$ or $\frac{\pi}{4}$

29. Given triangle ABC with $a = 7$, $C = 37^\circ$, and $B = 18^\circ$, find c . Round the answer to two decimal places.

$$\angle A = 180 - (37 + 18) = 125^\circ$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 125^\circ}{7} = \frac{\sin 37^\circ}{c}$$

$$c = \frac{7 \sin 37^\circ}{\sin 125^\circ}$$

$$c = 5.14$$

30. Given triangle ABC with $b = 8$, $c = 5$, and $A = 58^\circ$, find a . Round the answer to two decimal places.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 8^2 + 5^2 - 2(8)(5) \cos 58^\circ$$

$$a^2 = 46.6$$

$$a = 6.83$$

31. Solve triangle ABC given that $a = 12$, $b = 18$, and $c = 19$.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$19^2 = 12^2 + 18^2 - 2(12)(18) \cos C$$

$$361 = 144 + 324 - 432 \cos C$$

$$361 = 468 - 432 \cos C$$

$$-468 - 468$$

$$-107 = -432 \cos C$$

$$0.2477 = \cos C \quad C = 75.6^\circ$$

32. Write the equation of the resulting graph when $y = \sin x$ has an amplitude of 4, a period of π , is translated two units up, and shifted 5 units left.

$$y = 4 \sin 2(x + 5) + 2$$

$$b = 2$$

$$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$$

#31
 (cont)

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 75.6^\circ}{19} = \frac{\sin B}{18}$$

$$B = 66.6^\circ$$

$$A = 180 - (75.6 + 66.6)$$

$$A = 37.8^\circ$$